

Chi Square χ^2

Used for three different tests:

Test for Homogeneity of Proportions

Used to test if different populations have the same proportion of individuals with some characteristic.

Goodness of Fit

Used to test whether a frequency distribution fits an expected distribution.

Test for Independence

To test the independence of two variables. You can determine whether the occurrence of one variable affects the probability of the occurrence of the other variable.

Test for Homogeneity of Proportions

More than two parameters: $p_1, p_2, p_3, p_4, p_5, \dots, p_n$

1) Hypothesis

$$H_0: p_1 = p_2 = p_3 = \dots = p_n$$

H_1 : The population's proportions are not all equal

2) Collect data

3) To find p-value: $\chi^2 \text{cdf}(\chi^2, \infty, c-1)$ where $\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$

4) Decision: Reject H_0 if p-value is less than or equal to α

Note: If we reject the null hypothesis, then we can conclude that not all populations proportions are equal.

Example: Drug that reduces LDC (bad cholesterol)

	Group1	Group 2	Group 3	Row Total
Number of people who experience abdominal pain	51	5	16	72
Number of people who did not experience abdominal pain	1532	152	163	1847
Column Total	1583	157	179	1919

We want to know if the proportion of subjects in each group who experience abdominal pain is different at 1% significance level.

$$H_0: p_1 = p_2 = p_3$$

H_1 : The population's proportions are not all equal

Using a Calculator

```
MATRIX[A] 2 x3
[ 51      5      16 ]
[ 1532    152    163 ]
```

```
MATRIX[B] 2 x3
[ 59.393  5.8906  6.716 ]
[ 1523.6  151.11  172.28 ]
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```
χ2-Test
Observed: [A]
Expected: [B]
Calculate Draw
```

```
χ2-Test
χ2=14.70651321
P=6.4050309E-4
df=2
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By Hand

Expected Values $E_i = \frac{(\text{row total}) \cdot (\text{column total})}{\text{table total}}$

P-value

$\chi^2 cdf(\chi^2, \infty, df)$ where $df = \text{number of parameters} - 1$

Example Calculations:

$$E_1 = \frac{(72) \cdot (1583)}{1919} = 59.39 \quad E_2 = \frac{(1847) \cdot (1583)}{1919} = 1523.61 \quad \dots$$

$$\begin{aligned} \chi^2 &= \sum \frac{(O_i - E_i)^2}{E_i} \\ &= \frac{(51 - 59.39)^2}{59.39} + \frac{(1532 - 1523.61)^2}{1523.61} + \frac{(5 - 5.89)^2}{5.89} + \frac{(16 - 6.71)^2}{6.71} + \frac{(152 - 151.11)^2}{151.11} + \frac{(163 - 172.28)^2}{172.28} \\ &= 14.707 \end{aligned}$$

$$\chi^2 cdf(14.707, \infty, 2) = 0$$